Dynamic Earth Lab 6 – Hawaii Hotspot Lab

May 28, 2003

Abstract

In this lab we will try to understand the signature of a deep mantle plume in a classic hotspot track (Hawaii Hotspot). This track will allow us to compute the rate of plate motion with respect to the hotspot source (deep mantle). We assume through our calculations in here that the plates are flat and not spherical. This will facilitate the computations. On the other hand, we will attempt to infer the rate at which plume material is delivered to the base of the lithosphere, and what fraction is converted (melted) to magmas that can produce volcanoes, assuming that volcanoes has cone shape and by computing cross sectional areas of a topographic cross section across the Hawaiian swell.

1 Solution to exercises

1. **Relative plate motions.** We compute velocities of two plates, namely, Pacific and North American plates. For the Pacific plate, from (a) to (c), we use the Hawaiian hotspot track to compute its velocity with respect to the deep mantle.

   (a) Given the map of the Hawaiian-Emperor chain of volcanic islands we get the following measures:

$$10^\circ \text{ longitude} = 3.4 \text{ cm of ruler}$$
$$\text{Distance between Yuryaku Seamount and Necker} = 7.2 \text{ cm of ruler}$$

Since 10^4 km is approximately 90° of longitude, we get the following speed

$$\text{speed} = \frac{7.2 \text{ cm ruler} \times \frac{10^\circ}{3.4 \text{ cm ruler}} \times \frac{10^4 \text{ km}}{90^\circ} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{10^2 \text{ cm}}{1 \text{ m}}}{(43.4 - 10.3) \text{ Ma}} = 7.108 \, \frac{\text{cm}}{\text{year}}$$

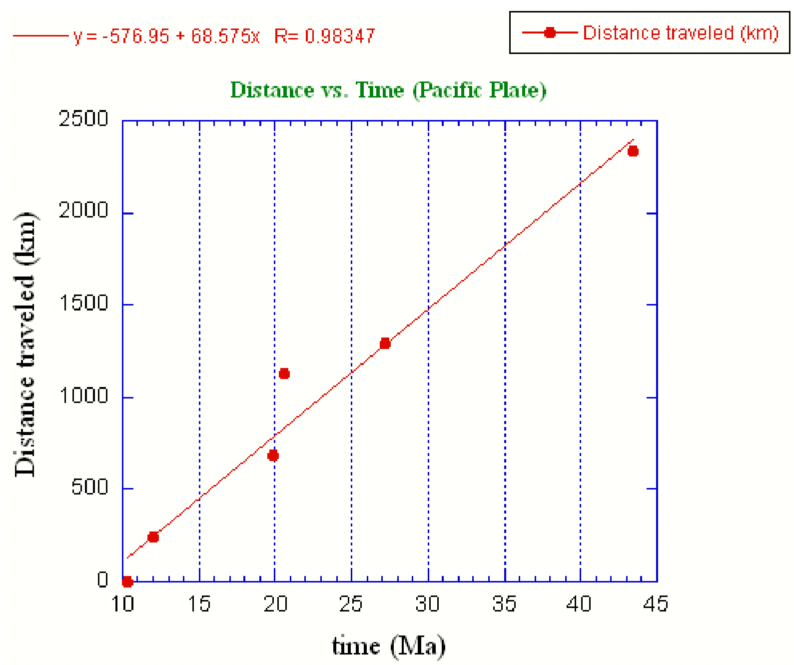
Now, for the angle we have

$$x = 6.6 \text{ cm ruler} \times \frac{10^\circ}{3 \text{ cm ruler}} \times \frac{10^4 \text{ km}}{90^\circ}$$
$$y = 2.8 \text{ cm ruler} \times \frac{10^\circ}{3.4 \text{ cm ruler}} \times \frac{10^4 \text{ km}}{90^\circ}$$

Then $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(0.374) = 20.5^\circ$. Hence

$$v_{pac/dm} = 7.108 \, \frac{\text{cm}}{\text{year}} \; 20.5^\circ \text{ N of W.}$$

(b) From the dated lavas documented at different points along the chain, we plot the distance traveled as a function of time.



From the plot, by fitting the points with a straight line we get a little more accurate plate speed:

$$68.575\,\frac{\text{km}}{\text{Ma}} = 6.8575\,\frac{\text{cm}}{\text{year}}.$$

Further, the speed has not been steady through time since the dots from the plot do not lie in a straight line.

(c) From the map given in the handout, we can deduce that the plume has been delivering material to the base of the lithosphere 59.6 Ma ago (in average).

(d) For the Yellowstone hotspot track in western North America we have that the speed is

$$\text{speed} = \frac{3 \text{ cm} \times \frac{2^\circ}{4.4 \text{ cm}} \times \frac{10^4 \text{ km}}{90^\circ} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{10^2 \text{ cm}}{1 \text{ m}}}{6 \text{ Ma} \times \frac{10^6 \text{ years}}{1 \text{ Ma}}} = 2.5\,\frac{\text{cm}}{\text{year}}$$

and the angle

$$\theta = \tan^{-1}(0.657) = 33.3^\circ.$$

Hence, the velocity of the N. American plate with respect to the deep mantle is

$$v_{nor/dm} = 2.5\,\frac{\text{cm}}{\text{year}}\; 33.3^\circ \text{ W of S}.$$

(e) We now calculate the angle and the speed of the North American plate with respect to the Pacific plate. From above, we got the following relative velocities

$$v_{nor/dm} = 2.5,\; 33.3^\circ \text{ W of S } = (-2.5\cos(33.3), -2.5\sin(33.3))$$
$$v_{pac/dm} = 7.1,\; 20.5^\circ \text{ N of W } = (-7.1\cos(20.5), 7.1\sin(20.5))$$

Then

$$
\begin{aligned}
v_{nor/pac} = v_{mor/dm} + v_{dm/pac} = v_{nor/dm} - v_{pac/dm} &= (-2.1, -1.4) - (-6.7, 2.5) \qquad (1)\\
&= (4.6, -3.9) \qquad (2)
\end{aligned}
$$

Thus,

$$\text{speed} = \sqrt{(4.6)^2 + (3.9)^2} \approx 6\theta = \tan^{-1}\left(\tfrac{3.9}{4.6}\right) = 40.2^\circ$$

Hence, $v_{nor/pac} = 6\,\frac{\text{cm}}{\text{year}}$ 40.2° S of E.

(f) San Andres fault is oriented roughly N35W. Since this fault is part of the boundary between the Pacific and North American plate and it is almost parallel to the relative velocity of the North American plate with respect to the Pacific one we have that San Andres is a transform fault boundary (right lateral). Quantitatively we have

$$
\begin{aligned}
v_{nor/san} = v_{nor/dm} - v_{san/dm} &= (-2.5\cos(33.3), -2.5\sin(33.3)) - (-7.1\sin(35), 7.1\cos(35))\\
&= (2, -7.2)
\end{aligned}
$$

Then

$$v_{nor/san} = 7.5\,\frac{\text{cm}}{\text{year}}\ 74.5^\circ\text{S of E.}$$

2. **Swell topography and volume.** Our calculations made reference to the graph of a topographic cross section across the Hawaiian swell provided in the handout.

(a) The approximation for the cross area of the swell is

$$A = 2000 \times 1 = 2000\ \text{km}^2$$

(b) Volume of the swell that is produced per unit time:

$$\phi_s = A\,v = 2000 \cdot 7 \times 10^{-5} = 0.14 \frac{\text{km}^3}{\text{year}}.$$

In here, v is the plate speed and we use a typical value for it.

(c) The plume volumetric flow rate can be computed by the formula

$$\phi_p = \phi_s \frac{\rho_m - \rho_w}{\rho_m \alpha \Delta T},$$

where typical values for upper mantle and water densities are 3300 kg/m^3 and 1000 kg/m^3 respectively, $\alpha = 3 \times 10^{-5}/^\circ C$, and $\Delta T = 300^\circ C$. Thus, the rate of delivery of plume material to the base of the lithosphere is

$$\phi_p = 1.84 \times 10^5 \frac{\text{km}^3}{\text{Ma}}.$$

3. **Volcanic Volume.** We already know from the above calculation the volume delivery rate to the base of the lithosphere. We want to know now what fraction of this melts. We do the following in order:

(a) Estimation of the volume of a volcano (Hawaii):

$$\text{Radius: } r = 1 \text{ cm ruler} \times \frac{2^\circ}{3.4 \text{ cm ruler}} \times \frac{10^4 \text{ km}}{90^\circ} = 65.36 \text{ km}$$

We approximate the volume of a volcano with the volume of a cone

$$V_{\text{volcano}} \approx V_{\text{cone}} = \frac{1}{3}\pi r^2 h, \quad \text{where } h = 8 \text{ km}$$
$$= 3.58 \times 10^4 \text{ km}^3$$

Now distance between volcanoes:

$$D = 2.5 \text{ cm ruler} \times \frac{2^\circ}{3.4 \text{ cm}} \times \frac{10^4 \text{ km}}{90^\circ} = 163.4 \text{ km}$$

By the way, I think that because of plumes erupt in discrete amount of time we do not have a continuous line of lava.

Then, the estimate of the average cross–sectional area of basaltic lava is

$$\frac{V_{\text{volcano}}}{D} = \frac{3.58 \times 10^4}{1.634 \times 10^2} = 2.2 \times 10^2 \text{ km}^2$$

(b) We compute the average of eruption rate, in volume per unit time:

$$\phi_e = \text{Cross sectional area of the volcanoes} \times \text{plate speed}$$
$$= 2.2 \times 10^2 \cdot 7.108 \times 10^{-5} = 1.56 \times 10^4 \tfrac{\text{km}^3}{\text{Ma}}$$

(c) We compare the eruption rate with the rate of delivery by the plume by calculating the ratio:

$$\frac{\phi_e}{\phi_p} = \frac{1.56 \times 10^4}{1.84 \times 10^5} = 8.5 \times 10^{-2}.$$

This ratio is a lower bound on the partial melt rate of plume material since not all the lava goes out from the volcano.