**EART162 Homework #5** **Due Tuesday 6th May 2008**

1. Say we want to establish a manned base on an asteroid which is rotating with a period of 10 hours. What thickness of insulating rock would we have to pile on the roof to avoid the extreme temperature variations caused by the rotation? Assume that the thermal diffusivity of rock is $10^{-6}$ $m^2s^{-1}$. (3 total)

2. Here we are going to reproduce an argument first used by Lord Kelvin in 1862 to constrain the age of the Earth. He used the observed, present-day heat flux, and the assumption that the Earth started off hot, to deduce how long it had been cooling.

a) Assume the Earth started off at a constant, hot temperature and then began to cool from the top. Write an *approximate* expression for $d$, the depth to which cooling has penetrated after time $t$. (1)

b) The surface temperature is $T_s$ and the temperature of the deep interior is $T_b$. Write down an expression for the heat flux $F$ as a function of $d$ (1)

c) Using these two results, write down an approximate expression for $t$ in terms of $T_b$,$T_s$,$F$, the thermal conductivity $k$ and the thermal diffusivity $\kappa$. (3)

d) Say the present-day heat flux is 40 $mWm^{-2}$, $\kappa$=$10^{-6}$ $m^2s^{-1}$, $k$=3 $Wm^{-1}K^{-1}$ and $T_b$-$T_s$=1000 K. How long has the Earth been cooling? (3)

e) This calculation was not seriously questioned for almost fifty years. Suggest two things wrong with Kelvin's analysis. (2) (10 total)

3. Here we're going to consider the effect of a core on the temperature distribution inside an internally-heated planet.

a) In steady state, the spherical heat conduction equation is

$$\kappa \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{H}{C_p}=0$$

Here $T(r)$ is the temperature, $r$ is the radial position, $\kappa$ is the thermal diffusivity, $H$ is the heat production rate in W/kg and $C_p$ is the specific heat capacity.
By integrating twice, find the general expression for $T$ (you will have two unknown constants). (4)

b) Let the planet have a core of radius $R_c$. At $r=R_c$, the temperature gradient is zero (there is no heat production in the core). Use this information to determine one of your constants of integration. (2)

c) The second boundary condition is that at the surface, $r=R$, the temperature is $T_s$. Use this information to determine your second boundary condition, and hence write down the complete solution for $T$. (4)

d) Verify that your solution reduces to the solution given in the notes if $R_c=0$. (1)

e) If $R_c=R/2$, write down an expression for the temperature at the core-mantle boundary in terms of $H, R, \kappa$ and $C_p$. (3)

f) Discuss how this temperature changes as you change $H, R$, $\kappa$ and $C_p$, and explain why each effect makes sense. (4)

g) For Ganymede, let's assume $H=5\times10^{-12}$ W kg$^{-1}$ $\kappa=10^{-6}$ m$^2$s$^{-1}$, $C_p=1000$ Jkg$^{-1}$K$^{-1}$, $T_s=100$ K and $R=2400$ km. If $R_c=R/2$, what is the temperature at the base of Ganymede's mantle? (2)

h) State whether you think the core of Ganymede is molten or solid, and suggest one way in which your conclusion could be tested by spacecraft observations (3) (23 total)