

**EART162 Homework #6****Due Tuesday 20<sup>th</sup> May 2008**

1. The Earth's mantle probably started life as a magma ocean. Here we'll make some rough estimates of its behaviour. You'll have to make reasonable guesses for some of the quantities involved; we are only worried about order-of-magnitude accuracy (at best) here. Be careful to state the values you are assuming.

a) As the magma ocean crystallizes, the crystals will sink to the bottom. Write down an order-of-magnitude expression for the viscous drag force that a crystal of radius  $r$  experiences when it is falling at a speed  $u$  through magma of viscosity  $\eta$  (1)

b) By balancing the drag force against the buoyancy force acting on the crystal, write down an expression for the terminal velocity of the crystal. You will need to use the density contrast between the crystal and the magma,  $\Delta\rho$ . (3)

c) For a crystal of radius 1 cm and density contrast  $100 \text{ kg m}^{-3}$ , how long would it take to reach the base of a magma ocean 1000 km deep if the viscosity is  $1000 \text{ Pa s}$ ? (2)

d) Using your answer to b), find an expression for the Reynolds number of the falling crystal in terms of  $r$  and the viscosity. How big would the crystal have to be in order for the inertial forces acting on the crystal to be comparable to the viscous forces? (4)

e) What heat transport mechanism controls how rapidly the magma ocean cools? What two factors mainly control whether a magma ocean on a smaller body cools more or less rapidly? (3)

f) What would happen to the cooling rate of the magma ocean if the crystals were less dense than the surrounding magma, and why? (2) (15 total)

2. Here we'll consider flow in a vertical cylindrical pipe. In the absence of pressure gradients, the 1-D Navier-Stokes equation in cylindrical coordinates is written

$$\frac{Dv}{Dt} = \eta \left[ \frac{\partial^2 v}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right) \right] - \Delta\rho g$$

Here  $v$  is the vertical velocity in the  $z$ -direction,  $r$  is the radial coordinate perpendicular to  $z$ ,  $\eta$  is the viscosity and  $\Delta\rho$  is the (constant) density contrast.

a) We'll consider steady-state flow in which the velocity doesn't change in the  $z$ -direction. Write down the N-S equation under these conditions (3).

b) Solve this equation to obtain the velocity profile  $v(r)$  for a pipe of radius  $R$ . To find the constants of integration you will need to apply two boundary conditions (7).

c) Say we have a volcanic conduit of radius 10 m and a basaltic lava of viscosity  $10^4$  Pa s and a density contrast with the surrounding rock of  $100 \text{ kg m}^{-3}$ . What is the maximum speed at which the lava will rise? (3)

d) If, in addition to the density contrast, there is also a pressure gradient of 2 MPa/km which is also forcing the lava upwards, by how much will the maximum velocity increase? (2) (15 total)

3. Europa has an ice shell floating on a subsurface ocean. Here we'll attempt to constrain the thickness of the ice shell.

a) Europa's radius and density are 1565 km and  $2900 \text{ kg m}^{-3}$ , respectively. What is the surface acceleration due to gravity? (2)

b) There are features on the surface of Europa which are thought to be caused by convection. If this is correct, what can we infer about the Rayleigh number of Europa's ice shell? (2)

c) Use the definition of the Rayleigh number and your answer to a) to determine the minimum thickness of the ice shell on Europa. Relevant parameter values for ice are: thermal expansivity  $10^{-4} \text{ K}^{-1}$ , temperature contrast 150 K, thermal diffusivity  $10^{-6} \text{ m}^2 \text{ s}^{-1}$ , viscosity  $10^{15} \text{ Pa s}$ , density  $1000 \text{ kg m}^{-3}$ . (4)

d) Why has the ocean beneath the ice shell not entirely frozen? (2) (10 total)