

**EART162 Homework #7****Due Tuesday 27<sup>th</sup> May 2008**

1. Here we're going to investigate the rigidity structure of Mars.

- a) The Sun is  $3 \times 10^6$  times more massive than Mars, and is 67,000 Mars radii distant from Mars. If the radius of Mars is 3400 km, what is the size of the equilibrium tide  $H$  raised by the Sun on Mars? (3)
- b) Spacecraft measurements show that the actual size of the solar tide on Mars is 0.75 cm. What is the Love number ( $h_2$ ) for Mars? (1)
- c) What gravity anomaly (in mGal) does a tidal bulge of 0.75 cm correspond to (assume a surface density of 3 g/cc)? (1)
- d) For uniform rigid objects, the tidal deformation is reduced by a factor of  $1 + \tilde{\mu}$ , where  $\tilde{\mu}$  is a measure of the importance of the rigidity  $\mu$  and is given by:

$$\tilde{\mu} = \frac{19}{2} \frac{\mu}{\rho g R} \quad (1)$$

Here  $\rho$  is the bulk density,  $g$  is the gravitational acceleration and  $R$  is the planetary radius. Rewrite equation (1) giving  $\tilde{\mu}$  in terms of  $\rho$ ,  $R$  and  $G$ , where  $G$  is the gravitational constant. (2)

- e) Using your answer to b) and your lecture notes, what is the observed value of  $\tilde{\mu}$  for Mars? (2)
- f) Using your answers to d) and e), what is the effective rigidity of Mars? You may assume that the bulk density of Mars is  $3900 \text{ kg m}^{-3}$  and  $G=6.7 \times 10^{-11}$  in SI units (3)
- g) The rigidity of near-surface rocks is typically 100 GPa. How do you explain your answer to f)? (3) (15 total)

2. Here we're going to investigate the history of the Earth-Moon system.

- a) Assuming that the Earth is a uniform sphere, write down an expression for the angular momentum of the Earth due to its rotation, where  $\omega$  is the rotation angular frequency,  $M$  is its mass and  $R$  its radius. (1)
- b) Assuming that the Moon is in a circular orbit, write down an expression for its orbital angular momentum, where  $m$  is the mass of the Moon,  $a$  is the Earth-Moon distance and  $G$  is the gravitational constant. (3)
- c) Hence calculate the total angular momentum of the Earth-Moon system (you can neglect the Moon's spin angular momentum). Take  $a=384,000 \text{ km}$ ,  $m=7 \times 10^{22} \text{ kg}$ ,  $M=6 \times 10^{24} \text{ kg}$  and  $R=6400 \text{ km}$ . (2)
- d) At an earlier time, the Moon was closer to the Earth (distance  $a'$ ) and the Earth was spinning faster (angular frequency  $\omega'$ ). Use your answers to a) and b) to write down an expression for  $\omega'$  in terms of  $a$ ,  $a'$ ,  $G$ ,  $M$ ,  $m$ ,  $\omega$  and  $R$ . State what assumption you are making (4).
- e) The Moon probably formed at a distance  $a'$  from the Earth, called the *Roche limit*, where the equilibrium tide  $H$  raised by the Earth on the Moon just equaled the radius of the Moon,  $R_m$ . Write down an expression for  $a'$  in terms of  $M$ ,  $m$  and  $R_m$  and hence calculate  $a'$  given that  $R_m=1800 \text{ km}$  (3).

- f) Using your answers to c), d) and e), what was the rotational period of the Earth when the Moon was at the Roche limit? (3)
- g) If the current recession rate of the Moon is 3.5 cm/yr, what is the rate at which energy is being added to the Moon's orbit (in W)? (4)
- h) How will the spin rate of the Earth evolve in the future? (2) (22 total)