9 April 08

ES162 PS#1

2 1a) $\frac{dM}{dt}$ $\left[\frac{kg}{s}\right]$ $\rho_s v R^2$ $\left[\frac{kg}{m^3} m s^{-1} m^2\right]$

4 b) $M = \frac{4}{3}\pi R^3 \rho \Rightarrow \frac{dM}{dt} = 4\pi R^2 \frac{dR}{dt}\rho$

$\frac{dM}{dt} = \rho_s v R^2 = 4\pi R^2 \rho \frac{dR}{dt} \Rightarrow \frac{dR}{dt} = \frac{1}{4\pi}\frac{\rho_s}{\rho} v$

$\Rightarrow \int_0^{R_0} dR = \int_0^{\tau} \frac{1}{4\pi}\frac{\rho_s}{\rho} v \, dt \Rightarrow \tau = 4\pi R_0 \rho / \rho_s v$

2 c) $\tau = \frac{4\pi R_0 \rho}{\rho_s v}$ $v = 25\times10^3$ $\rho_s = 10^{-9}$ $\rho = 4\times10^3$ $\tau = 191$ Ma

13 5 d) $\frac{\rho_{s1}}{\rho_{s2}} = \left(\frac{a_1}{a_2}\right)^{-2.5} \Rightarrow \rho_s(\text{Neptune}) = 8.8\times10^{-13}\ kg\,m^{-3}$

$\tau = \frac{4\pi R_0 \rho}{\rho_s v} = 870$ Gyr

This is much longer than the age of the solar system! One possibility is that Neptune formed closer in (where ρ_s + v are larger), and moved out later.

2 2 a) 124.7 g per mole. Half a mole of Si

2 b) 60.1 g per mole, Half a mole of SiO_2.

3 c) Fe = 55.8 g per mole. FeS = 87.9 g per mole.

0.45 moles of FeS. 0.44 moles of Fe

6 d) Mantle = $0.5\times124.7 + 0.5\times60.1 = 92.4$

Core = $0.45\times87.9 + 0.44\times55.8 = 64.1$

Mantle mass fraction 59% core mass fraction 41%

15 2 e) Some of the iron is contained in the mantle not the core

1 3 a) $6GM^2/5R$

1 b) $V_t = \frac{4}{3}\pi R^3 + \frac{4}{3}\pi R^3 = \frac{8}{3}\pi R^3 = \frac{4}{3}\pi R_t^3$

$\Rightarrow R_t^3 = 2R^3 \Rightarrow R_t = (2)^{1/3} R$

1 c) $\frac{3G(2M)^2}{5(2^{1/3})R} = \frac{3}{5}\frac{GM^2}{R} \cdot 2^{5/3}$

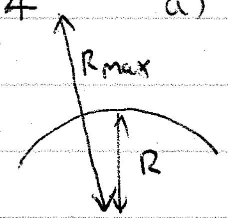
ES162 PS#1 contd.

5 2 d) $\Delta E = \frac{3}{5}\frac{GM^2}{R}\left(2^{5/3} - 2\right) = \frac{3}{5}\frac{GM^2}{R}(1.17)$

This is comparable to the binding energy of an individual body.

Conclusion: a lot of energy is given off during the collision.

3 4 a)



final $PE = -\frac{GMm}{R_{max}}$ initial $PE = -\frac{GMm}{R}$

difference is due to KE: $\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{R_{max}}$

$\therefore v = (2GM)^{1/2}\left(\frac{1}{R} - \frac{1}{R_{max}}\right)^{1/2}$

2 b) If $R_{max} \to \infty$ $v = \left(\frac{2GM}{R}\right)^{1/2}$ = escape velocity.

3 c) $v = (2GM)^{1/2}\left(\frac{R_{max}-R}{R_{max}R}\right)^{1/2} \approx (2GM)^{1/2}\left(h/R^2\right)^{1/2}$

$g = \frac{GM}{R^2} \Rightarrow v = (2gh)^{1/2}$

This makes sense - we are equating KE ($\frac{1}{2}mv^2$) with PE (mgh).

10 2 d) $\frac{v^2}{2GM} = \frac{1}{R} - \frac{1}{R_{max}} \Rightarrow R_{max} = 3550$ km $\Rightarrow$ antipodal material gets 150 km off surface

43