15 April 08

ES 162 HWK #2

1\. a) $\frac{GM}{a^2} = a\omega^2 \quad \omega = \frac{2\pi}{P} = 1.5\times10^{-5}\,s^{-1} \quad a = 1180\,m$

4 $\Rightarrow M = a^3\omega^2/G = 5.7\times10^{18}$ kg

2 b) $M = \frac{4}{3}\pi r^3 \rho \Rightarrow \rho = 1100$ kg m^{-3}

7 1 c) $\rho \ll \rho_{silicate} \Rightarrow$ either highly porous or contains a lot of ice!

2 a) 

$I_1 = 0.4MR^2 = 0.4\,\frac{4}{3}\pi R^5 \rho_m$

$\Delta\rho = \rho_c - \rho_m$

$\rho_c-\rho_m$ — R_c $\quad I_2 = 0.4MR_c^2 = 0.4\,\frac{4}{3}\pi R_c^5 \Delta\rho$

$I_{total} = I_1 + I_2 = 0.4\left(\frac{4}{3}\pi\right)\{R^5\rho_m + R_c^5\Delta\rho\}$

$M_{total} = \frac{4}{3}\pi\left(R^3\rho_m + R_c^3\Delta\rho\right)$

$$\frac{I_{total}}{M_{total}R^2} = \frac{0.4\left(\frac{4}{3}\pi\right)\left(R^5\rho_m + R_c^5\Delta\rho\right)}{\left(\frac{4}{3}\pi\right)R^2\left(R^3\rho_m + R_c^3\Delta\rho\right)} = \frac{0.4R^5\rho_m\left(1+\left(\frac{R_c}{R}\right)^5\frac{\Delta\rho}{\rho_m}\right)}{R^2R^3\rho_m\left(1+\left(\frac{R_c}{R}\right)^3\frac{\Delta\rho}{\rho_m}\right)}$$

$$\Rightarrow \frac{I_{tot}}{M_{tot}R^2} = \frac{0.4\left(1+\left(\frac{R_c}{R}\right)^5\frac{\Delta\rho}{\rho_m}\right)}{\left(1+\left(\frac{R_c}{R}\right)^3\frac{\Delta\rho}{\rho_m}\right)}$$

7\.

2 b) if $R_c = 0$ or $\Delta\rho = 0$ then $I/MR^2 = 0.4$, as required

c) $\Delta\rho = 5000 \quad R = 3500 \quad R_c = 2000 \quad \rho_m = 3000$

11 2 $I/MR^2 = 0.4\left(\frac{1.10}{1.31}\right) = 0.336$

2 3 a) uniform $\Rightarrow \frac{C}{Ma^2} = 1 \Rightarrow \Theta = \left(\frac{25}{4}\right)\left(1-\frac{3}{2}\frac{2}{5}\right)^2 = \left(\frac{25}{4}\right)\left(\frac{2}{5}\right)^2 = 1$

b) $f = \frac{3}{2}J_2 + \frac{1}{2}\frac{a^3\omega^2}{GM}$

2 $J_2 = \frac{2}{3}f - \frac{1}{3}\frac{a^3\omega^2}{GM} = 0.0149$

c) assume hydrostatic!

$$\Theta = \frac{\frac{5}{2}\frac{\omega^2a^3}{GM}}{f_{hyd}} - 1 = \frac{25}{4}\left(1-\frac{3}{2}\frac{C}{Ma^2}\right)^2$$

$$\Rightarrow \left(\frac{4}{25}\right)^{1/2}\left\{\frac{\frac{5}{2}\frac{\omega^2a^3}{GM}}{f_{hyd}} - 1\right\}^{1/2} = 1 - \frac{3}{2}\frac{C}{Ma^2}$$

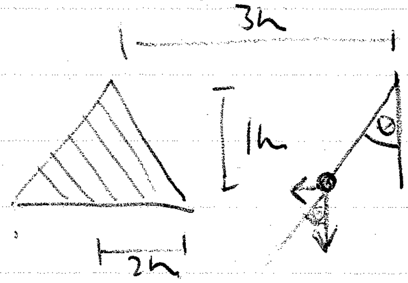
4 $\Rightarrow 0.68 = 1 - \frac{3}{2}\frac{C}{Ma^2} \Rightarrow \frac{C}{Ma^2} = 0.21$

HWK #2 contd.

2 3 d) Density contrasts in gas giants are much larger than in silicate bodies ⇒ gas giants can be more centrally concentrated (lower MoI)

1 e) if $\rho_c \gg \rho_m$ then $\frac{C}{MR^2} \approx 0.4 \left(\frac{R_c}{R}\right)^2$

11 — f) It turns out it is impossible to get $\frac{C}{MR^2} = 0.21$ for a two layer model (!). I did not mark this part of the question.



2 4 a) $V = \frac{1}{3}\pi r^2 h$ $M = 1.05 \times 10^{13}$ kg

6 b) $mg \sin\theta = \frac{GMm}{r^2}$ $\sin\theta \approx \theta$ (radians !)

$\therefore G = \frac{g \theta r^2}{M} = 7.3 \times 10^{-11}$ in SI units

10 2 c) G is slightly too large - either we have overestimated r or we have underestimated M. Probably the latter - maybe there is dense material at depth or our shape model is wrong?

$7 + 11 + 11 + 10 = 39$ total