PS #5 13 May 08.

1. $t \sim d^2/K \Rightarrow d \sim (Kt)^{1/2} \sim 0.19\,m$ (3).

2. a) $d \sim (Kt)^{1/2}$ (1)

b) $F = \frac{k(T_b - T_s)}{d}$ (1).

c) $\frac{k(T_b - T_s)}{F} = (Kt)^{1/2} \Rightarrow t = \frac{k^2(T_b - T_s)^2}{F^2 K}$ $\frac{W^2 m^{-2} K^{-2} K^2}{W^2 m^{-4} m^2 s^{-1}} = s$ ✓ OK (3)

d) 180 Myr (3)

e) (1) He assumed conduction (not convection)

(2) He didn't know about radioactive decay. (2) [10]

3. a) $\frac{K}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{H}{C_p} = 0 \Rightarrow \frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{Hr^2}{KC_p}$

$\Rightarrow r^2\frac{dT}{dr} = -\frac{Hr^3}{3KC_p} + a \Rightarrow \frac{dT}{dr} = -\frac{Hr}{3KC_p} + \frac{a}{r^2}$

$\Rightarrow T = -\frac{Hr^2}{6KC_p} - \frac{a}{r} + b$ a, b unknown constants (4).

b) $r = R_c$ $\frac{dT}{dr} = 0 \Rightarrow -\frac{HR_c}{3KC_p} + \frac{a}{R_c^2} = 0 \Rightarrow a = \frac{HR_c^3}{3KC_p}$ (2)

c) $r = R$ $T = T_s \Rightarrow T_s = -\frac{HR^2}{6KC_p} - \frac{HR_c^3}{3KC_pR} + b$

$\Rightarrow b = T_s + \frac{HR^2}{6KC_p} + \frac{HR_c^3}{3KC_pR}$

$\Rightarrow T = -\frac{Hr^2}{6KC_p} - \frac{HR_c^3}{3KC_pr} + T_s + \frac{HR^2}{6KC_p} + \frac{HR_c^3}{3KC_pR}$

$= \frac{H}{6KC_p}(R^2 - r^2) + \frac{HR_c^3}{3KC_p}\left\{\frac{1}{R} - \frac{1}{r}\right\} + T_s$ (4).

PS#5 contd.

d) If $R_c = 0$ then $T = T_s + \frac{H}{6KC_p}(R^2 - r^2) = T_s + \frac{H\rho}{6k}(R^2 - r^2)$ ✓OK (1)

e) $R_c = R/2 \Rightarrow T = \frac{H}{6KC_p} R^2 \left\{1 - \frac{1}{4}\right\} + \frac{HR^3}{3 \cdot 8KC_p} \left\{\frac{1}{R}\right\} \left\{1 - \frac{2}{1}\right\} + T_s$

$$= \frac{HR^2}{KC_p}\left\{\frac{1}{6}\cdot\frac{3}{4} - \frac{1}{24}\right\} + T_s = \frac{HR^2}{12KC_p} + T_s \quad (3)$$

f) H↑ T↑ – more radioactive heating causes higher temperatures

R↑ T↑ – smaller surface area : volume ratio ⇒ harder for heat to escape

K↑ T↓ – higher conductivity means smaller temperature gradients are required to get the heat out

$C_p$↑ T↓ – more energy required to change temperature (4)

g) 2500 K (2)

h) Probably molten. If there is a molten core, we can look for a dynamo using a magnetometer

NB Measuring the MoI will reveal a core but not whether it is solid or liquid. (3)

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