EART 162 #6 22 May 08

1a) $F_d \sim \eta r u$ ①

b) $F_b \sim r^3 \Delta\rho g \Rightarrow u \sim \frac{r^2 \Delta\rho g}{\eta}$ $\frac{Pa\, m}{Pa\, s}$ ③

c) $u \sim 10^{-4}\, ms^{-1}$

$\Rightarrow t \sim 300$ yrs ②.

d) $Re = \frac{\rho L u}{\eta}$ here $L \sim r \Rightarrow Re \sim \rho \frac{r^3 \Delta\rho g}{\eta^2}$

if $Re \sim 1$ then $r \sim 0.7$m (assuming $\rho = 3000\, kg m^{-3}$) ④

e) Advection – sinking crystals remove cold material and allow hot material to reach the surface (where it cools by radiation).

Smaller body has smaller volume : surface area ratio => cools faster.

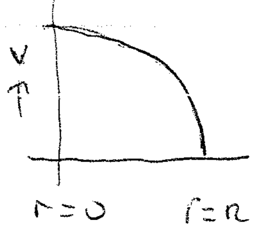
③ But g is also smaller, so crystals sink more slowly => slower cooling.

f) The crystals would float to the surface and form an insulating crust through which the heat would have to be conducted → slower cooling ② 15

2a) $\eta \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial v}{\partial r} = \Delta\rho g$ ③

b) $\frac{\partial}{\partial r} r \frac{\partial v}{\partial r} = \frac{\Delta\rho g r}{\eta} \Rightarrow r \frac{\partial v}{\partial r} = \frac{\Delta\rho g r^2}{2\eta} + a$

$\Rightarrow \frac{\partial v}{\partial r} = \frac{\Delta\rho g r}{2\eta} + \frac{a}{r} \Rightarrow v = \frac{\Delta\rho g r^2}{4\eta} + a \ln r + b$



$v = 0$ at $r = R$ $\frac{\partial v}{\partial r} = 0$ at $r = 0 \Rightarrow a = 0$

$\frac{\Delta\rho g R^2}{4\eta} + b = 0 \Rightarrow b = -\frac{\Delta\rho g R^2}{4\eta}$

$$\Rightarrow \quad v = \frac{\Delta \rho g}{4\eta}(R^2 - r^2)$$ (7)

c) $V_{max} = \frac{\Delta \rho g R^2}{4\eta} = 2.5\ ms^{-1}$ (3)

d) $v_{max} = \frac{R^2}{4\eta}\left(\Delta \rho g + \frac{dP}{dz}\right)$ $\Delta \rho g = 1\ MPa/km$

$\Rightarrow$ velocity will increase by a factor of 3 (2)   [15]

3 a) $g = \frac{GM}{R^2} = G\frac{4}{3}\pi \frac{R^3 \rho}{R^2} = 1.27\ ms^{-2}$ (2)

b) $Ra \geq Ra_{crit} \approx 1000$ (2)

c) $Ra = \frac{\rho g \alpha \Delta T d^3}{\kappa \eta} \geq 1000 \Rightarrow d \geq \left(\frac{1000\,\kappa \eta}{\rho g \alpha \Delta T}\right)^{1/3}$

$\therefore$ $d \geq 3.7$ km. (4)

d) Tidal heating in the ice shell keeps the ocean from freezing. Maybe also tidal heating in the silicate interior? (2).

[10]