EART 162 #7 1st June 08

1(a) $H = R\left(\frac{m}{M}\right)\left(\frac{R}{a}\right)^3$ $\frac{M}{m} = 3\times10^6$ $\frac{R}{a} = \frac{1}{67{,}000}$ $H = 0.034\,m$ (3)

b) $d = h_2 H \Rightarrow h_2 = 0.22$ (1)

c) $0.75\times10^{-5}\times3\times42 = 0.00095$ mGal (1)

d) $g = \frac{GM}{R^2} = G\frac{4}{3}\pi R\rho$ $\tilde{\mu} = \frac{57\mu}{8G\pi R^2\rho^2}$ (2)

e) uniform body $h_2 = \frac{5}{2}\frac{1}{1+\tilde{\mu}}$ $h_2 = 0.22 \Rightarrow \tilde{\mu} = 10.4$ (2)

f) $\mu = \frac{8G\pi R^2\rho^2\tilde{\mu}}{57} = 54$ GPa (3)

g) Effective rigidity of Mars is less than that of (near-surface) rocks. This suggests that some part of Mars has a low rigidity; most likely explanation is a liquid core. (3) $\boxed{15}$

2 a) angular momentum $= I\omega = 0.4MR^2\omega$ (1)

b) $h = mna^2$ $n^2 = GM/a^3 \Rightarrow n = (GM/a^3)^{\frac{1}{2}}$

$\Rightarrow I = m(GMa)^{1/2}$ (3)

c) Earth $= 7.15\times10^{33}$ Moon $= 2.74\times10^{34}$

total $= 3.46\times10^{34}$ kg m^2 s^{-1} (2)

d) Conservation of angular momentum.

$0.4MR^2\omega_1 + m(GMa_1)^{1/2} = 0.4MR^2\omega_2 + m(GMa_2)^{1/2}$

$(\omega_2-\omega_1)0.4MR^2 = m(GM)^{1/2}(a_1^{1/2}-a_2^{1/2})$

$\Rightarrow \omega_2-\omega_1 = \frac{m(GM)^{1/2}}{0.4MR^2}(a_1^{1/2}-a_2^{1/2})$

$\Rightarrow \omega_2 = \omega_1 + \frac{mG^{1/2}}{0.4M^{1/2}R^2}(a_1^{1/2}-a_2^{1/2})$ (4).

e) $H = \frac{M}{m}\left(\frac{R_m}{a_2}\right)^3 R_m \Rightarrow \frac{M}{m}\left(\frac{R_m}{a_2}\right)^3 = 1 \Rightarrow a_2 = \left(\frac{M}{m}\right)^{1/3} R_m$

$\Rightarrow a_2 = 4.4 R_m = 7940\,\text{km}$ (3).

f) $\omega_2 = \omega_1 + \frac{mG^{1/2}}{0.4 M^{1/2} R^{2}}(a_1^{1/2} - a_2^{1/2})$ $a_1 = 384\,000$ km $\omega_1 = 7.27\times10^{-5}$

$\Rightarrow \omega_2 = 3.12\times10^{-4} \Rightarrow$ period $= 5.6$ hrs (3).

g) energy $= -\frac{GMm}{2a} = E$ $\frac{dE}{dt} = \frac{dE}{da}\cdot\frac{da}{dt} = \frac{GMm}{2a^2}\cdot\frac{da}{dt}$

$\Rightarrow \frac{dE}{dt} = 10^{11}\,\text{W}$ (4).

h) It will continue to slow until the Earth's spin rate equals the Moon's orbital period (synchronous). (2) $\boxed{22}$.