EART162 MIDTERM 4 May 2008

1 a) H, O, Si, S (2)

b) $\sigma = E\varepsilon \Rightarrow \sigma = 5\ MPa$ (1)

c) $g_0 = \frac{GM}{r^2}$ $g' = \frac{GM}{(r+h)^2} = \frac{GM}{r^2(1+\frac{h}{r})^2} \approx \frac{GM}{r^2}(1 - \frac{2h}{r})$

$g' - g_0 = -g_0 \frac{2h}{r}$ (3)

d) $H_{tot} = \frac{4}{3}\pi R^3 H$ $F = \frac{H_{tot}}{4\pi R^2} = \frac{1}{3} RH = 11.7\ mWm^{-2}$ (2)

e) $F = \frac{k\Delta T}{d} \Rightarrow d = 175\ km$ (2)

f) $\frac{GM}{r^2} = r\omega^2$ $\omega_1^2 r_1^3 = \omega_2^2 r_2^3 \Rightarrow r_2 = r_1 \left(\frac{\omega_1}{\omega_2}\right)^{2/3}$ $\omega_2 = \frac{1}{2}\omega_1$

$\Rightarrow r_2 = 670{,}000\ km.$ (2)

g) uncompensated ($\lambda \ll \alpha$) (2)

h) $42\ mGal/km/g\,cc^{-1}$ say $\rho = 3\ gcc^{-1}$, $\Delta g \approx 250\ mGal$ (1)

i) $d \sim \sqrt{\kappa t}$ ~~the~~ $\kappa = 10^{-6}\ ms^{-1} \Rightarrow d \sim 2.3m$ (2)

j) $g(r) = \frac{GM}{r^2} = \frac{G}{r^2}\frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi G r \rho$

$dP = -\rho g\, dr = -\frac{4}{3}\pi G r \rho^2\, dr = -\frac{4}{3}\pi G \rho^2 \left[\frac{r^2}{2}\right] + c$

at $r = R$, $P = 0 \Rightarrow P = \frac{2}{3}\pi G \rho^2 (R^2 - r^2) \Rightarrow P_{cen} = \frac{2}{3}\pi G \rho^2 R^2$ (3)

(20 total)

2 a) The bulk modulus is the pressure change required to cause the initial density of the material to double; or the bulk modulus controls the fractional change in density for a given change in pressure (2)

b) $\frac{d\rho}{\rho} = \frac{dP}{K}$ $dP = \rho g\, dz \Rightarrow \frac{d\rho}{\rho} = \frac{\rho g}{K} dz \Rightarrow \int \frac{1}{\rho^2} d\rho = \int \frac{g}{K} dz$

$\Rightarrow -\frac{1}{\rho} + c = \frac{gz}{K}$ now at $z=0$ $\rho = \rho_0 \Rightarrow \frac{1}{\rho_0} - \frac{1}{\rho} = \frac{g}{K} z$ (5)

c) $\frac{1}{\rho} = \frac{1}{2\rho_0} \Rightarrow \frac{1}{2\rho_0} = \frac{gz}{K} \Rightarrow z = \frac{K}{2\rho_0 g}$ (2)

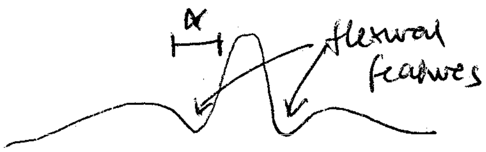
d) $z = \frac{K}{2\rho_0 g} = 1850\,km$ (2)

e) We assumed g is constant, whereas in reality it will decrease with depth. So we ~~[illegible]~~ overestimate the pressure and underestimate the depth. (2)

f) Determine the flattening (or the polar gravity (J_2)) and get the moment of inertia by making the hydrostatic assumption. Or look for a dynamo (like the Earth or Mercury). (2)

(15 total)

3 a)



$\alpha \approx 150\,km$ (2)

b) $\alpha = \left(\frac{4D}{(\rho_m-\rho_w)g}\right)^{1/4}$ $D = \frac{E\,T_e^3}{12(1-\sigma^2)}$ $\alpha = \left(\frac{4E\,T_e^3}{12(1-\sigma^2)\Delta\rho g}\right)^{1/4}$ $T_e = \left(\frac{3\alpha^4(1-\sigma^2)\Delta\rho g}{E}\right)^{1/3}$

$\Rightarrow T_e = 68\,km$ (5)

c) $F = \frac{h\Delta T}{d} \Rightarrow \frac{\Delta T_1}{d_1} = \frac{\Delta T_2}{d_2}$ (assuming F constant) $\Rightarrow d_2 = \frac{\Delta T_2}{\Delta T_1} d_1 = 99\,km$. (4)

d) $t \sim \frac{d^2}{K}$ $K = 10^{-6}\,m^2 s^{-1}$ $\Rightarrow t \sim 310\,Myr$. (2)

e) Convective upwelling (plume) can give rise to long-wavelength swells (2)

(15 total)

4 a) By measuring the pole-to-equator gravity variation we can determine $J_2 \left(= \frac{C-A}{MR^2}\right)$. If we make the hydrostatic assumption, we can use the Darwin-Radau equation to determine C from $C-A$. If we can also measure the precession rate of the rotation axis, this gives us $\frac{C-A}{C}$ which we can combine with J_2 to get C directly. (6)

b) $dI = \frac{1}{2}\pi r^4 \rho\, dz$ (1)

c) $dI = \frac{1}{2}\pi r^4 \rho\, dz = \rho\frac{1}{2}\pi(R^2-z^2)^2 dz \Rightarrow I = \rho\frac{1}{2}\pi\int_{-R}^{R}(R^2-z^2)^2 dz$

$I = \frac{\rho}{2}\pi\left[R^4 z - \frac{2z^3}{3}R^2 + \frac{z^5}{5}\right]_{-R}^{R} = \frac{8}{15}\pi R^5 \rho$ (5)

d) $M = \frac{4}{3}\pi R^3 \rho$ $I/MR^2 = \frac{2}{5}$ (1)

e) Gas giants undergo much larger changes in density from the surface to the centre (because gases are more compressible than solids). So the degree of central condensation is larger and the MoI smaller. (2)

(15 total)