

Note

Forced obliquity and moments of inertia of Titan

Bruce G. Bills^{a,b,*}, Francis Nimmo^c^a NASA Goddard Space Flight Center, Greenbelt, MD 20771-001, USA^b Scripps Institution of Oceanography, Institute of Geophysics and Planetary Physics, La Jolla, CA 92093-0225, USA^c Department of Earth and Planetary Science, University of California, Santa Cruz, CA 95064, USA

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ABSTRACT

The obliquity of Titan is small, but certainly non-zero, and may be used to place constraints on Titan's internal structure. The measured gravity coefficients of Titan imply that it is non-hydrostatic and thus the normal Darwin–Radau approach to determining internal structure cannot be applied. However, if the obliquity is assumed to be tidally damped (that is, in a Cassini state) then combining the obliquity with the measured gravity coefficients allows Titan's moment of inertia to be determined without invoking hydrostatic equilibrium. For polar moment values in the range ($0.3 < C/MR^2 < 0.4$), tidally-damped obliquity values of ($0.115^\circ < |\epsilon| < 0.177^\circ$) result. If the inferred moment value exceeds 0.4, this strongly suggests the presence of a near-surface ice shell decoupled from the interior, probably by a subsurface ocean.

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1. Introduction

There is great interest in determining the interior structure, and specifically the moments of inertia, of satellites in the outer Solar System. Conventionally, this is done by using spacecraft flybys to measure the degree-2 gravity coefficients, and then deriving the moment of inertia by assuming that the satellites are hydrostatic and have no long-term rigidity (Hubbard and Anderson, 1978; Rappaport et al., 1997). Here we present a complementary approach, in which the moments of inertia may be obtained by measuring both the obliquity, or angular separation of the spin and orbit poles, and the degree-2 gravity coefficients. As we describe below, this approach has the advantage of not requiring an assumption of hydrostatic equilibrium. However, it does require an alternative assumption concerning the extent to which the obliquity has been damped, and that the body is precessing rigidly (see below). In practice, the two techniques can be used to determine independent estimates of the moments of inertia; the degree to which these estimates agree provides a check on whether the assumptions made were appropriate.

2. Moments from gravity

In a spherical harmonic expansion of the gravitational potential, there are 5 independent coefficients of harmonic degree 2, but there are 6 independent terms in the inertia tensor (Soler, 1984). If the coordinate axes are chosen to align with principal inertial axes, the inertia tensor diagonalizes, three of the potential coefficients vanish and the remaining two gravity coefficients, which we denote J_2 and $C_{2,2}$, have a simple relationship to the principal moments $A < B < C$

$$\begin{bmatrix} J_2 \\ C_{2,2} \end{bmatrix} MR^2 = \begin{bmatrix} C - (A+B)/2 \\ (B-A)/4 \end{bmatrix}, \quad (1)$$

where M and R are the mass and mean radius of the body, respectively.

If an external gravitational potential Φ_j , with spatial pattern represented by a spherical harmonic of degree j , is imposed upon an initially spherical body, it will result in deformation of that body, and will give rise to an additional induced

potential Ψ_j , which will also share the spatial pattern of a degree j harmonic. The ratio k_j of the imposed and induced potentials, evaluated at the surface, is defined to be a Love number (Munk and MacDonald, 1960),

$$\Psi_j[\theta, \phi] = k_j \Phi_j[\theta, \phi] \quad (2)$$

and reflects the resistance of the body to deformation. For a homogeneous, incompressible elastic body, with mean radius R , density ρ and elastic rigidity μ , the degree two tidal Love number is

$$k_2 = \frac{3}{2} \left(\frac{2\sigma}{2\sigma + 19\mu} \right), \quad (3)$$

where

$$\sigma = \rho g R = \frac{4\pi G}{3} \rho^2 R^2 \quad (4)$$

is an effective gravitational rigidity. If the potential is applied for long enough, it is expected that the elastic stress will relax, and the effective Love number will revert to that due to density alone. This is known as a secular Love number.

An isolated, non-rotating fluid body will attain an equilibrium configuration of spherical symmetry. If such a fluid (hydrostatic) body is subjected to tidal and rotational potentials, it will deviate from spherical symmetry by an amount which is diagnostic of the internal density structure. In particular, for a synchronous hydrostatic rotator the combined tidal and rotational deformation will be (Hubbard and Anderson, 1978)

$$\begin{bmatrix} J_2 \\ C_{2,2} \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix} \frac{qk_s}{12}, \quad (5)$$

where ω is the rotation angular frequency, G is the gravitational constant and the strength of the perturbation is

$$q = \frac{\omega^2 R^3}{GM} \quad (6)$$

and the resistance to deformation is parameterized by k_s , the secular Love number.

If J_2 and $C_{2,2}$ are both known, we have two independent estimates of k_s . However, it is difficult to measure both J_2 and $C_{2,2}$ from a flyby, as J_2 is best assessed from a polar orbit and $C_{2,2}$ from an equatorial orbit. The approach used for the Galilean satellites (Anderson et al., 1996, 1998a, 1998b, 2001) is to invoke the hydrostatic ratio of $J_2/C_{2,2} = 10/3$ and estimate a single lumped parameter.

* Corresponding author at: Scripps Institution of Oceanography, Institute of Geophysics and Planetary Physics, La Jolla, CA 92093-0225, USA.

E-mail address: bbills@ucsd.edu (B.G. Bills).

Once the secular Love number is known, it can be used in the Darwin–Radau relation (Radau, 1885; Darwin, 1899)

$$c \equiv \frac{C}{MR^2} = \frac{2}{3} \left(1 - \frac{2}{5} \sqrt{\frac{4 - k_s}{1 + k_s}} \right) \quad (7)$$

to estimate the polar moment of inertia. This relation holds exactly only for uniform density fluids, but is a reasonably good approximation for radially stratified bodies (Nakiboglu, 1982). This approach, which requires the hydrostatic assumption, is at present the only source of information about moments of inertia of the Galilean satellites. This relation is easily inverted to yield

$$k_s = \frac{-3(5c - 6)(15c - 2)}{225c^2 - 300c + 116}. \quad (8)$$

3. Moments from precession

Moments of inertia of a body are most fundamentally rooted in rotational dynamics, and it is clearly true that assumptions of hydrostatic equilibrium are not always required. For rapidly rotating bodies, like Earth or Mars, the rate of precession of the spin pole provides information which complements the J_2 and $C_{2,2}$ estimates, and allows a determination of the moments of inertia without assuming hydrostatic equilibrium. The precession is a response to gravitational torques. For unit vectors \hat{s} and \hat{n} , along the spin and orbit pole directions, respectively, the spin pole precession is given by (Ward, 1973; Kinoshita, 1977)

$$\frac{d\hat{s}}{dt} = \alpha(\hat{n} \cdot \hat{s})(\hat{s} \times \hat{n}) \quad (9)$$

and the rate parameter is given by

$$\alpha = \frac{3}{2} \frac{n^2}{\omega} \left(\frac{C - (A + B)/2}{C} \right) (1 - e^2)^{-3/2}, \quad (10)$$

where n and e are the orbital mean motion and eccentricity. Assuming that we know the numerical values of the other parameters (n, ω, e), the equations for α, J_2 , and $C_{2,2}$ can be solved for all three principal moments (A, B, C) without requiring any assumption of hydrostatic equilibrium. For Mars, the precession rate is roughly 10 arcsec/year (Folkner et al., 1997). For Earth, the rate is much higher, roughly 50 arcsec/year (Williams, 1994), partly because of greater proximity to the Sun, but roughly 2/3 of the torque comes from the Moon.

Venus is an interesting example, in that, though there are numerous published estimates of the moment of inertia (Yoder, 1995; Correia and Laskar, 2001, 2003; Correia et al., 2003) all are based upon analogy with Earth, rather than direct observational constraints. Observing the spin pole precession of Venus will be difficult, both because the spin pole is nearly anti-aligned with the orbit pole, and because the precession is expected to be slow, as a result of the relatively small departure of Venus from spherical symmetry.

4. Moments from obliquity

We present an alternative strategy for estimating moments of inertia of synchronous satellites. As noted above, gravitational torques from the primary will make a satellite spin pole precess about its orbit pole. If the orbit pole itself were inertially fixed, then dissipation would drive the obliquity to zero. However, if the orbit pole is also precessing then the tidally damped configuration is somewhat more intricate. The simplest case is that in which the orbit pole precesses at a uniform rate about an inertially fixed direction, or invariable pole. In such a configuration, known as a Cassini state, the spin pole adjusts its angular separation from the orbit pole so that it remains coplanar with the orbit pole and invariable pole, as the former precesses about the latter. Though it was noted by G.D. Cassini, in 1693, that the Moon behaves this way, a proper understanding of the dynamics came much later. Colombo (1966) analyzed the dynamics of a uniformly precessing oblate spheroid, and Peale (1969, 1974) extended the analysis to cover the case of a triaxial ellipsoid. Some of the interesting behavior associated with capture into these states has been subsequently explored (Ward, 1975; Gladman et al., 1996; Quinn et al., 1997; Touma and Wisdom, 1998; Wisdom, 2006).

The condition for this co-planar precession, in nearly circular orbits, can be written as (Ward, 1975)

$$(v + (u - v) \cos[\varepsilon]) \sin[\varepsilon] = \sin[i - \varepsilon], \quad (11)$$

where i is the inclination of the orbit pole to the invariable pole, and ε is the obliquity or separation of spin and orbit poles. The parameters u and v are related to the moments of inertia of the body, and the relative rates of orbital motion and orbital precession.

The first of these parameters has the form

$$u = Up \quad (12)$$

where the moment dependent factor is

$$U = \frac{3}{2} \left(\frac{C - A}{C} \right) = \frac{3}{2} \left(\frac{J_2 + 2C_{2,2}}{c} \right). \quad (13)$$

The relative rates of orbital motion and orbit plane precession is

$$p = \frac{n}{d\Omega/dt}, \quad (14)$$

where n is the mean motion, and Ω is the longitude of the ascending node of the orbit. For most bodies, the node regresses and this ratio is thus negative. The second parameter has a similar factorization

$$v = Vp \quad (15)$$

with

$$V = \frac{3}{8} \left(\frac{B - A}{C} \right) = \frac{3}{2} \left(\frac{C_{2,2}}{c} \right). \quad (16)$$

When these substitutions are made, the constraint equation (11) can be written in either of the alternative forms

$$2c \sin[i - \varepsilon] = 3p(C_{2,2} + (J_2 + C_{2,2}) \cos[\varepsilon]) \sin[\varepsilon], \quad (17)$$

$$8C \sin[i - \varepsilon] = 3p(B - A + (4C - B - 3A) \cos[\varepsilon]) \sin[\varepsilon]. \quad (18)$$

These constraint equations are linear in polar moment, but nonlinear in obliquity. Thus, if values for all the other parameters were known, we could rather trivially solve for the polar moment as

$$c = \frac{3p}{2} \left(\frac{(C_{2,2} + (J_2 + C_{2,2}) \cos[\varepsilon]) \sin[\varepsilon]}{\sin[i - \varepsilon]} \right), \quad (19)$$

$$C = \frac{3p}{2} \frac{(B - A - (3A + B) \cos[\varepsilon]) \sin[\varepsilon]}{4 \sin[i - \varepsilon] - 3p \sin[2\varepsilon]}. \quad (20)$$

When solving these constraint equations for obliquity, the situation is somewhat more subtle. In general, there are either two or four distinct real solutions for obliquity, depending upon the values of the input parameters. In all cases, the spin pole \hat{s} , orbit pole \hat{n} , and invariable pole \hat{k} are coplanar. It is also convenient to define a signed obliquity, with positive values corresponding to \hat{s} and \hat{n} on opposite sides of \hat{k} . Following Peale (1969), the usual numbering of these separate Cassini states $\{S_1, S_2, S_3, S_4\}$ is that S_1 is \hat{s} near to \hat{k} and on the same side as \hat{n} ; S_2 is somewhat farther from \hat{k} , and on the opposite side from \hat{n} ; S_3 is retrograde, and thus nearly antiparallel to \hat{n} ; and S_4 is on the same side of \hat{k} as S_1 , but farther from \hat{n} and \hat{k} . These spin states represent tangential intersections of a sphere (possible orientations of the spin pole) and a parabolic cylinder representing the Hamiltonian.

If the radius of curvature of the parabola is too large, there are only two possible spin states, otherwise there are four. At the transition point, states 1 and 4 coalesce and vanish. In the axi-symmetric case, for which $v = 0$, the transition occurs at (Henrard and Murigande, 1987; Ward and Hamilton, 2004)

$$u = -(\sin[i]^{2/3} + \cos[i]^{2/3})^{3/2} \quad (21)$$

and

$$\tan[\varepsilon] = -\tan[i]^{1/3}. \quad (22)$$

If the magnitude of the parameter u is larger than the value given by Eq. (21), then all four Cassini states exist.

All four of the Cassini states represent equilibrium configurations. That is, if the spin pole \hat{s} is placed in such a state, it will precess in such a way as to maintain a fixed orientation relative to \hat{n} and \hat{k} . The states S_1, S_2 , and S_3 are stable, in the sense that small departures from equilibrium will lead to finite amplitude librations. Each of these states is the dynamical center of a domain of stable librations, and these three domains cover the entire sphere. In contrast, S_4 is unstable. We will argue below that Titan's obliquity is tidally damped, and that it is most likely to occupy the S_1 state.

In the present context, we are only interested in relatively small values for inclination and obliquity. Following Ward and Hamilton (2004), we can rewrite the constraint equation (11) in the form

$$(\cos[i] + v + (u - v) \cos[\varepsilon]) \tan[\varepsilon] = \sin[i]. \quad (23)$$

For small inclinations, the right-hand side will be small. We can make the left-hand side small either by taking $\tan[\varepsilon]$ small (which implies $\cos[\varepsilon] \rightarrow 1$), or by making the term in parentheses small. In the first case, we obtain

$$\varepsilon = \tan^{-1} \left[\frac{\sin[i]}{1 + u} \right] \quad (24)$$

which approximates Cassini state 1. In the second case, the result is

$$\varepsilon = \cos^{-1} \left[\frac{v + \cos[i]}{v - u} \right] \quad (25)$$

which approximates Cassini state 2.

In the limit of small angles, we can use Eq. (24) to write the linearized relationship between inclination and obliquity as

$$\frac{i}{\varepsilon} = 1 + u = 1 + \left(\frac{3p}{2c}\right)(J_2 + 2C_{2,2}). \quad (26)$$

Note that this relation does not require the assumption of hydrostatic equilibrium. If, however, we invoke the hydrostatic relationship for J_2 and $C_{2,2}$, in our estimates of u and v , we find that the linearized connection between inclination and obliquity becomes

$$\frac{i}{\varepsilon} = 1 + \frac{2k_s pq}{c}. \quad (27)$$

If we knew the gravity coefficients (J_2 , $C_{2,2}$) and the angles i and ε , we could form estimates of the polar moment in several different ways. Without the obliquity value, we could use the Darwin–Radau relation [Eq. (7)] with k_s estimates derived from J_2 only, $C_{2,2}$ only, or from both together [Eq. (5)]. If we had obliquity and only one of the gravity parameters, the polar moment could be estimated by making the hydrostatic assumption and using Eq. (27). If, however, both gravity parameters and the obliquity are known, then Eq. (26) can be used to estimate the polar moment without having to make the hydrostatic assumption. On the other hand, this approach does require that the obliquity is damped.

5. Application to Titan

The obliquity of Titan is not presently known with sufficient accuracy for use in estimating the moment of inertia. We need to clarify our nomenclature. Several recent studies of the radiative environment of Titan (Flasar, 1998; Roos-Serote, 2005; Tokano and Neubauer, 2005) have referred to the angular separation between the spin pole of Titan and the orbit pole of Saturn as Titan's obliquity. This angle, approximately equal to the 26.73° dynamical obliquity of Saturn, is certainly the relevant angle for consideration of radiative input to the atmosphere of Titan. That is due to the circumstance that Titan's orbit plane lies nearly in Saturn's equator plane.

However, from an orbital and rotational dynamics perspective, the important angle is the much smaller, and currently unknown, separation between the spin pole of Titan and the pole of its own orbit about Saturn. This angle is not well known, at present, for several reasons. First is that the surface of Titan is obscured by a substantial atmosphere, and even the conclusive establishment of a synchronous rotation rate for Titan only occurred relatively recently (Lemon et al., 1995; Richardson et al., 2004). Second is that the angle is small; almost certainly less than one degree, as we will demonstrate below. However, it appears quite likely that the radar images from the Cassini mission, when they begin to provide overlapping coverage of Titan's surface (Elachi et al., 2004), will provide dramatically improved estimates of the rate and direction of spin.

The expectation that Titan's obliquity will be tidally damped requires some justification. Peale (1976) examined the time scales for damping of free wobbles and librations which might have been excited on the Moon, and the analysis is largely applicable to Titan. The major uncertainty is that the rate of tidal dissipation within Titan is not well constrained. However, for any plausible rheology, the damping time scale will be less than 10⁶ yr. An additional complication at Titan is the presence of a substantial atmosphere. It has been proposed that angular momentum exchange between the solid body and atmosphere could cause the instantaneous solid-body rotation rate to deviate slightly from synchronous (Tokano and Neubauer, 2005). Analogous meridional effects could tilt the spin pole away from the plane containing the orbit pole and invariable pole. Seasonal variations in orientation of Earth's spin pole are largely driven by atmospheric and oceanic flow (Wahr, 1982).

The nominal processing of Cassini radar images assumes that the obliquity is zero. The angular resolution of the images depends on encounter geometry, but is typically of order 100 m. The angular resolution for obliquity determination is set by the ratio of radar resolution and radius of the satellite (2575 km). We thus expect that a resolution of order 0.01° should be achievable by requiring overlapping radar images to be properly registered.

Equation (26) shows that J_2 and $C_{2,2}$ are required to calculate the damped obliquity. Though the hydrostatic assumption has been made for the interpretation of the Galilean satellite gravity field estimates (Anderson et al., 1996, 1998a, 1998b, 2001), there is ample precedent for departure from that pattern in similar sized objects. The gravity coefficients J_2 and $C_{2,2}$ need not necessarily be in their hydrostatic ratio of 10:3. For example, the lunar ratio is roughly 9:1 (Zuber et al., 1994). A recent analysis of Doppler tracking data from three flybys of Titan by the Cassini spacecraft yield estimates of the degree two harmonic coefficients (less et al., 2007)

$$J_2 = (27.22 \pm 0.19) \times 10^{-6}, \quad (28)$$

$$C_{2,2} = (11.16 \pm 0.05) \times 10^{-6}. \quad (29)$$

These values are very far from the expected pattern for hydrostatic equilibrium. In the hydrostatic case, the ratio $J_2/C_{2,2}$ has the value 10/3. In contrast to that, the observed ratio is only 73% of the hydrostatic value. This implies that the moment of inertia of Titan cannot be inferred from the Darwin–Radau relation.

A significant number of internal structure models of Titan have been presented (Grasset and Sotin, 1996; Grasset et al., 2000; Sohl et al., 2003; Tobie et al., 2005a, 2005b) and despite the lack of observational constraints on the moments of inertia, a plausible range of values for the polar moment is likely

$$0.3 \leq c < 0.4 \quad (30)$$

with the upper bound corresponding to a homogeneous interior. We now estimate the range of damped obliquity values that this range of c would imply.

The orbital contributions to the obliquity expressions can be easily evaluated. The inclination of the orbit, relative to Saturn's equator plane, is 0.28°. Titan's orbit plane precesses, mainly in response to torques from the oblate figure of Saturn, and with a minor solar contribution (Sinclair, 1977; Harper and Taylor, 1993). The nodal regression period is 607.56 years, with a corresponding node rate of

$$\frac{d\Omega}{dt} = -0.5925^\circ/\text{year}. \quad (31)$$

The orbital period of Titan is 15.95 days, and the mean motion is thus

$$n = 22.577^\circ/\text{day}. \quad (32)$$

The ratio of these rates, which occurs in the factors u and v , is

$$p = \frac{n}{d\Omega/dt} = -1.391 \times 10^4. \quad (33)$$

With these values of input parameters, Titan is readily seen to be in the regime where all 4 Cassini state are possible.

If we simply incorporate the observed values of J_2 and $C_{2,2}$ into the Cassini state constraint equation (11), we obtain

$$c = \frac{(0.2329 + 0.8008 \cos[\varepsilon]) \sin[\varepsilon]}{\sin[\varepsilon - i]}. \quad (34)$$

Fig. 1 shows the range of polar moment values implied by a range of obliquity values. The obliquity values are small and negative, corresponding to Cassini state S_1 . We note that, due to the values of the parameters p , q , and i , all 4 of the possible Cassini states exist for Titan, but the only small obliquity state available to Titan is S_1 . The obliquities in states S_2 and S_4 are nearly $\pm 90^\circ$, and that in state S_3 is very near to 180° .

If Titan were a homogeneous rigid body, the polar moment value would be $c = 2/5$, and the corresponding Cassini state obliquity would be $\varepsilon = -0.177^\circ$. Smaller angular separations between the spin and orbit pole would imply smaller polar moment values. For example, the value of $c = 0.3$, which is near the lower bound from published structure models, would imply an obliquity of $\varepsilon = -0.115^\circ$. This homogeneous value might then be considered a plausible upper bound on the magnitude of Cassini state obliquities. Larger values would imply either that the spin pole is not damped, or that the polar moment is in excess of that from a homogeneous sphere (see below).

However, as several internal structure models for Titan posit an internal fluid layer and a relatively thin shell (Grasset and Sotin, 1996; Grasset et al., 2000; Sohl et al., 2003; Tobie et al., 2005a, 2005b) it is perhaps worthwhile to consider the case in which the shell is dynamically decoupled from the interior. The observed gravity coefficients (J_2 and $C_{2,2}$) obviously apply to the entire body, and are not sensitive to the mechanical state of the interior.

In a simple model of a decoupled shell, it might be assumed to have uniform thickness h , and be floating on a fluid interior whose upper surface is an equipotential. The masses of the shell and entire satellite are

$$\delta M = \frac{4\pi}{3} \rho_s R^3 (1 - z^3), \quad (35)$$

$$M = \frac{4\pi}{3} (\rho) R^3 \quad (36)$$

where the normalized radius of the bottom of the shell is written as

$$z = \frac{r_s}{R} = 1 - h/R \quad (37)$$

and r_s is the mean radius of the bottom of the shell, ρ_s is the shell density, R and (ρ) are the satellite mean radius and density. The moment of inertia of the shell can similarly be represented as

$$\delta C = \frac{8\pi}{15} \rho_s R^5 (1 - z^5). \quad (38)$$

If the moment of inertia of the shell is written in terms of the mass of the shell itself, we obtain

$$\delta C = \frac{2}{5} \delta M R^2 \left(\frac{1 - z^5}{1 - z^3} \right). \quad (39)$$

The gravitational potential coefficients $G_{n,m}$ of a body which is composed of concentric shells, each of which has uniform density and topographic undulations on its nominally spherical surface can be easily computed. If the density contrasts between a shell and the region above it is $\Delta\rho_j$, the mean bounding radii are $r_j =$

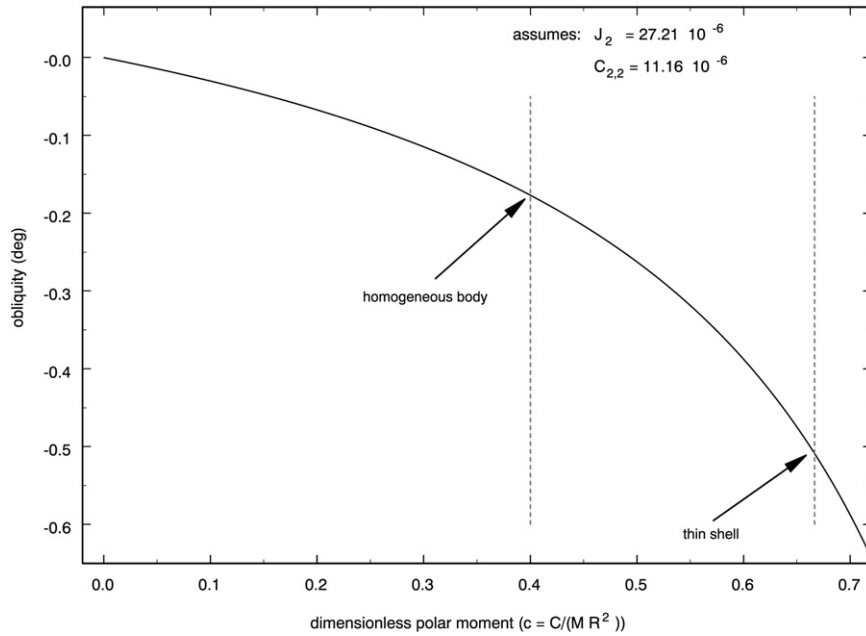


Fig. 1. Variation in forced obliquity of Titan as a function of normalized polar moment [Eq. (34)]. The polar moment controls the ratio of obliquity to inclination, and the inclination is assumed constant at 0.28° . Solid line is calculated assuming recently observed values of J_2 and $C_{2,2}$. Dashed vertical lines indicate locations of homogeneous sphere and thin shell values of the polar moment. The negative sign of the obliquity simply implies that the spin and orbit poles are on opposite sides of the invariable pole, as is appropriate for Cassini state S_1 . See text for discussion.

$\xi_j R$, and the surface topography spherical harmonic coefficients are $H_{n,m}^j$ then the potential is given by (Balmino, 1994; Chambat and Valette, 2005)

$$\langle \rho \rangle G_{n,m} = \left(\frac{3}{2n+1} \right) \sum_j \Delta \rho_j \xi_j^{n+3} H_{n,m}^j. \quad (40)$$

If the ice shell of Titan is floating on a fluid substrate, and has uniform thickness, the potential of the shell alone would be proportional to that of the entire body, and would scale according to

$$\delta G_{n,m}^* = \left(\frac{3}{2n+1} \right) \frac{\rho_s}{\langle \rho \rangle} (1 - z^{n+3}) G_{n,m} \quad (41)$$

if expressed as a fraction of the entire satellite monopole term. If instead we express the gravitational coefficients of the shell as fractions of the shell mass, we obtain

$$\delta G_{n,m} \equiv \delta G_{n,m}^* \frac{M}{\delta M} = \frac{3}{2n+1} \left(\frac{1 - z^{n+3}}{1 - z^3} \right) G_{n,m}. \quad (42)$$

The pertinent values for harmonic degree 2 are

$$\begin{bmatrix} \delta J_2 \\ \delta C_{2,2} \end{bmatrix} = \frac{3}{5} \left(\frac{1 - z^5}{1 - z^3} \right) \begin{bmatrix} J_2 \\ C_{2,2} \end{bmatrix}. \quad (43)$$

Note that the shell thickness scaling factor here is identical to that for the moment of inertia (39), as both are degree two moments of the mass distribution.

If we now substitute the shell values for gravity coefficients (43) and dimensionless polar moment of inertia (39) into the Cassini state constraint equation (17), we see that the shell thickness scaling factors cancel out and we obtain

$$4 \sin[i - \varepsilon] = 9p(C_{2,2} + (C_{2,2} + J_2) \cos[\varepsilon]) \sin[\varepsilon] \quad (44)$$

which is exactly equivalent to employing the thin shell moment of inertia value

$$C = \frac{2}{3} \delta M R^2. \quad (45)$$

If we use this version of the constraint equation, and estimate the obliquity implied by the observed values of the parameters J_2 , $C_{2,2}$, i , and p , we find

$$\varepsilon = -0.508^\circ. \quad (46)$$

This is not a rigorous upper bound on the magnitude of the obliquity, because it ignores the fact that, even with a fluid layer intervening, there would still be a gravitational torque coupling the shell to the deeper interior (Szeto and Xu, 1997), and the strength of that torque is difficult to constrain. However, we can plausibly argue that, absent the shell influence, the nominally solid parts of the deeper interior would be rotating in a Cassini state with obliquity corresponding to the gravity coefficients and moment of inertia of that solid part alone. This would be close to the homogeneous ($c = 2/5$) obliquity value of -0.115° . The coupling between shell and core would move the spin poles of each toward that of the other.

We note that the present situation for Titan has similarities to that for Mercury. In the case of Mercury, the degree two gravity field is only rather poorly constrained (Anderson et al., 1987), but the obliquity and amplitude of forced librations (Margot et al., 2007) strongly imply that an outer shell is decoupled from the core (Peale, 1972; Peale et al., 2002). When the gravity field is better determined from orbiting spacecraft, it will be possible to even better constrain the internal structure. It is to be anticipated that future spacecraft missions to Titan will examine the forced librations there as well.

6. Conclusions

When the obliquity of Titan is measured, it will provide further information about the internal structure of the body. The observed gravity field has already demonstrated that the body is not in hydrostatic equilibrium with the imposed tidal and rotational potentials, and as a result the Darwin–Radau relation cannot be invoked to estimate the polar moment of inertia. If the body is assumed to rotate rigidly, and to have its spin damped into a Cassini state, then the obliquity will allow estimation of the moment of inertia. As a necessary (but not sufficient) condition for testing the latter assumption, if the body does occupy a Cassini state, its spin pole will be coplanar with the orbit pole and Saturn's spin pole. If the inferred moment value exceeds the homogeneous value of $C = 2/5MR^2$, it will strongly suggest that a thin near-surface layer is mechanically decoupled from the deeper interior and is precessing somewhat independently.

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